Optimum Fracture Design

A fracture design has been placed in front of you. Is it the right answer for an optimum completion? How do you know? Is there too much, too little, or the correct amount of proppant? Is proppant strength correct for the expected closure pressure? Will the chosen fluid have enough viscosity to give the desired height or length? The goal of this newsletter is to empower you to the characteristics of a properly designed fracture treatment. Two terms need to be defined for this empowerment. The first is **Dimensionless Fracture Conductivity**, and the second is **Folds of Increase**.

**Dimensionless Fracture Conductivity**

Dimensionless fracture conductivity \( F_{cd} \) is defined as fracture conductivity, \( k_w \) (md-ft), divided by reservoir permeability \( k \) times fracture half-length, \( x_f \) (ft) (see Equation 1). It provides a means of optimizing the amount of conductivity in a fracture for varying permeability and fracture length.

\[
F_{cd} = \frac{k_w}{k x_f} \quad (1)
\]

It can be shown mathematically that for pseudo-radial & pseudo-steady-state conditions, the optimum value for well productivity occurs at \( F_{cd} \) of about 2. For a given amount of proppant, two different types of fractures can be generated, a short radius fracture can be created with a high value of \( k_w \), or a longer, narrow fracture can be created with a lower value of \( k_w \). Fracpacks in high permeability zones (>1 md) deal with the short radius fractures with a high \( k_w \), and in low permeability zones (<1 md), a long, lower conductivity fracture is desired.

For example, a partially depleted 50’ thick reservoir with 20 md and areal extent of 40 acres \( (r_e = 660') \) has a 50’ frac half-length. To achieve \( F_{cd} = 2 \), a \( k_w \) of 2,000 md-ft is needed. If this same zone has a permeability of 100 md, the \( k_w \) required would be 10,000 md-ft to yield the same \( F_{cd} \) of 2.

**What Is “Folds of Increase”?**

“Folds of Increase”- FOI is related to “Productivity Index”- PI in a ratio of fractured versus natural completion. The FOI is calculated by determining the flow rates before and after a fracture stimulation treatment or FOI= \( Q_f/Q_{nat} \). FOI can be calculated by using Equation 2

\[
FOI = \frac{\ln(r_e/r_w)}{\ln(r_e/r_w^\prime) + s} \quad (2)
\]

where \( r_e \) is drainage radius, \( r_w \) is wellbore radius, “s” is prefrac skin, and \( r_w^\prime \) is the “equivalent wellbore radius” as discussed below. Values for FOI can vary from 1 to 10, meaning no stimulation, to values > 10 for being very stimulated.

To calculate FOI, a term called “Effective Wellbore Radius”- \( r_w^\prime \) must be defined (after Prats). \( r_w^\prime \) can be described as if a wellbore has been under-reamed to an area equivalent to the area created from the frac as seen in Figure 1. Several methods have been developed to calculate \( r_w^\prime \). One method to calculate the “equivalent wellbore diameter” from \( F_{cd} \) is by using Figure 2 developed by Cinco-Ley. The relation described in Figure 2 is the heart of all fracturing for all but the lowest permeability formations. When \( F_{cd} \) is less than 0.5, \( r_w^\prime = 0.28 x k_w/k \), so the length doesn’t matter and the use of more or better proppant is required. Conversely, when \( F_{cd} \) is > 20, \( r_w^\prime = x_f/2 \), the fracture is acting like an infinite conductivity fracture and the use of better (stronger) proppants would be a waste of money. Using the previous 20 md example, for \( F_{cd} = 2 \), \( r_w^\prime/x_f \) equals 0.3. The \( x_f \) of 50’ x 0.3 yields \( r_w^\prime \) of 15’. FOI= \( \ln(660/0.5)/\ln(660/15) \) (assuming \( s=0 \)) or a FOI of 1.90, almost doubling production.

**FOI Sensitivity for Reservoirs**

One method for quick determination of the optimum fracture half-length and conductivity is running FOI sensitivity as seen in Figure 3. This does not give the best economic solution, but the best production solution. Note the FOI increases with length and conductivity. Thus, there is no single solution that produces optimum performance. But what is evident is a frac-length of 40-60 feet with 5-8 pounds/square feet (9,000-14,000 md-ft) appears to give the best results with FOI between 1.6 to 1.8.
**Effect of Different Proppants**

The conductivity associated with different proppants varies with the proppant, and with the proppant stress of the formation as seen in Figure 4. If Fcd calculated is very low, investigate increasing the concentration pounds per square feet or using higher strength proppant, which would increase the Fcd. Conversely, if Fcd is very high, a weaker (cheaper) proppant might be investigated to see the effect on FOI.

**How is Fcd Related to Production?**

Post-frac production is affected by the “Skin” near the wellbore. The more negative the skin, the lower the reservoir drawdown, and the higher the production rate. Conversely, with higher positive skin, the additional pressure drop near the wellbore reduces reservoir drawdown, and reduces rate. Fcd can be related directly to theoretical “Skin”, that is the skin excluding wellbore jewelry. Theoretical Skin can be calculated from Equation 3 by using the Effective Wellbore Radius. In the 20 md example above, skin would be \(-\ln(15/0.5)\) or \(s = -3.4\).

\[
s = -\ln\left(\frac{r_s}{r_w}\right)
\]  

(3)

Post fracture Skins from pressure build-up tests have been plotted for various areas around the world in Figure 5. It can be seen as Fcd increases, skin decreases. If the Fcd is below 2, the fracture can be conductivity limited, meaning the reservoir permeability can deliver more production than the propped fracture can carry to the wellbore. Using “Folds of Increase” can optimize treatments to avoid this.

**Fracture Transient Flow**

The discussion above was for wells with post-frac production dominated by pseudo-radial flow behavior. In such cases, productivity can be predicted based on a “Skin”, or an “Equivalent Wellbore Radius”, and the maximum productivity achievable with any proppant volume is for a combination of xf and propped width (kfw) giving Fcd about = 2.

For low permeability zones (< 1/100’s of md) and longer fractures (xf of 100’s of feet or more), fracture transient flow (bi-linear & linear flow) becomes important. For these cases, Fcd = 2 simply becomes a minimum design goal, and an “optimum” Fcd will be >2 (sometimes >> 2). A goal of Fcd = 10 is often used; however there is NO simple, single design value for such cases! The best procedure is to couple reservoir simulation & fracture modeling to determine proper design goals.

When will transient flow be important? An approximate answer comes from the definition of Dimensionless Time, \(t_{Dxf}\). Fracture transient (bi-linear/linear) flow ends, and pseudo-radial flow is seen as \(F_{cd}\) increases, skin decreases. If the \(F_{cd}\) is below 2, the fracture can be conductivity limited, meaning the reservoir permeability can deliver more production than the propped fracture can carry to the wellbore. Using “Folds of Increase” can optimize treatments to avoid this.

\[
t_{Dxf} = \frac{r_w^2}{k} \cdot \frac{F_{cd}}{\mu \phi C_p} \cdot \frac{C_i}{1/1000} \cdot \frac{x_f}{x_f} \approx 2
\]  

(4)

**Non-Darcy Flow**

Another case requiring additional conductivity is when non-darcy flow becomes important. As with effects of fracture transient flow, there are no simple, single design goals. The additional kfw required should be determined with an appropriate reservoir simulator.

**Conclusions**

Two of the most important factors for an optimum fracture design are Dimensionless Fracture Conductivity (Fcd) and “Folds of Increase”. The Fcd for an optimum fracture should be a minimum of 2. Whether permeability of your zone is 0.01 md and a long, low conductivity fracture is desired, or 100 md where a short wide fracture is needed, Fcd applies (though the optimum Fcd may vary from = 2 for moderate/high permeability formations, to > 2 for “tight” rock). Another investigating tool is “Folds of Increase”. FOI is a simple but very powerful design tool for all but the lowest of permeability zones.